Int. J. Heat Mass Transfer. Vol. 7, pp. 1145–1146. Pergamon Press 1964. Printed in Great Britain

ON THE TRANSITION FROM BLACK-BODY TO ROSSELAND FORMULATIONS IN OPTICALLY THICK FLOWS

C. L. TIEN and RALPH GREIF

University of California, Berkeley, California

(*Received* 9 *January* 1964)

THE PRESENT note is to extend the interesting idea put forward by Goulard [1] on the transition in optically thick flows from the black-body formulation to the Rosseland diffusion formulation. A smooth transition is achieved by employing: 1. a temperature-jump boundary condition as suggested by many investigators [2, 3, 4] to improve the diffusion approximation and 2. a matching procedure between the optically thin and optically thick regimes proposed by Probstein [4]. In analogy with the simple mean-free-path treatment of rarefied gases [5], the temperature-jump boundary conditions is assumed to be of the first order, i.e. the discontinuity being proportional to the temperature gradient of the gas extrapolated to the wall. The proportionality constant is then determined by requiring that the solution reduces to the black-body result in the limit of the optically thin regime.

For inviscid incompressible flow over a flat plate the energy equation is given by [1]

$$\rho_\infty u_\infty C_p \frac{\partial T}{\partial x} = \frac{16\,\sigma T_i^3}{3k_{Ri}} \frac{\partial^2 T}{\partial y^2} \tag{1}$$

where the volumetric absorption coefficient has been assumed to vary as a third power of the temperature

$$k_R/k_{Ri} = (T/T_i)^3 \tag{2}$$

The same notation is used as in reference 1. The boundary conditions are

$$\begin{aligned} T &= T_i && \text{for } x = 0, y > 0 \\ T &= T_w + C\,(\partial T/\partial y) && \text{for } x \geqslant 0, y = 0 \\ T &= T_i && \text{for } x \geqslant 0, y \to \infty \end{aligned} \tag{3}$$

where the first-order slip boundary condition has been imposed with a constant C to be determined later. The solution to equations (1) and (3) for the radiative heat flux to a cold wall ($T_w \ll T_i$) for large optical length τ can then be written in the form

$$q^R(0) = -\left(\frac{16Bo}{3\pi\tau}\right)^{1/2} \sigma T_i^4 \left(1 + \frac{3C^2}{16\tau} k_{Ri}^2\, Bo\right)^{-1/2} \tag{4}$$

where

$$Bo \equiv \text{Boltzmann number} = \frac{\rho_\infty u_\infty C_p T_i}{\sigma T_i^4} \tag{5}$$

$$\tau \equiv \text{optical length} = k_{Ri} x \tag{6}$$

Matching equation (4) with the black-body result in the limit of small optical length

$$q^R(0) = -\sigma T_i^4 \tag{7}$$

yields a value for C of $16/3k_{Ri}\sqrt{(\pi)}$. Thus, equation (4) becomes

$$q^R(0) = -\frac{16\,\sigma T_i^4}{3}\left(\frac{3Bo}{16\pi\tau}\right)^{1/2}\left(1 + \frac{16Bo}{3\pi\tau}\right)^{-1/2} \tag{8}$$

It is noted that extending equation (4) for small values of τ can not be theoretically justified. However, as pointed out by Probstein [4], similar extrapolations for radiation, for simultaneous radiation and conduction and for rarefied gas analysis are in good agreement with the correct numerical solutions and with experiments. Equation (8) is shown in Fig. 1 along with the black-body and the Rosseland results.

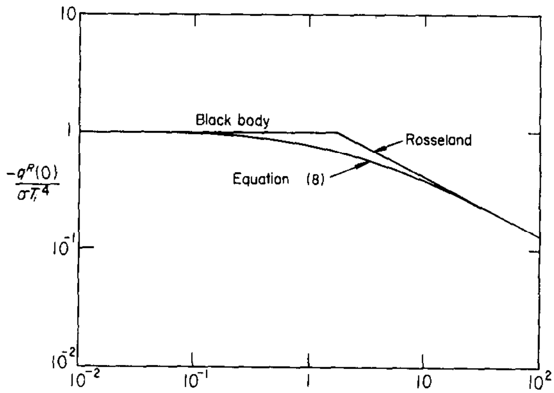

FIG. 1. Radiant heat flux in the inviscid incompressible flat-plate flow.

For axially symmetric inviscid incompressible stagnation flow, the velocity field is given by

$$u = ax, \qquad w = -2az \tag{9}$$

where a is some characteristic velocity gradient V/L. The energy equation is of the form [1]

$$2z\frac{\mathrm{d}T}{\mathrm{d}z} = -\frac{16\sigma T_i^3 L}{3\rho\, C_p\, V k_{Ri}} \frac{\mathrm{d}^2T}{\mathrm{d}z^2} \tag{10}$$

The boundary conditions are

$$T = T_w + C\,(\mathrm{d}T/\mathrm{d}z) \text{ for } z = 0$$
$$T = T_i \qquad \text{for } z \to \infty \qquad (11)$$

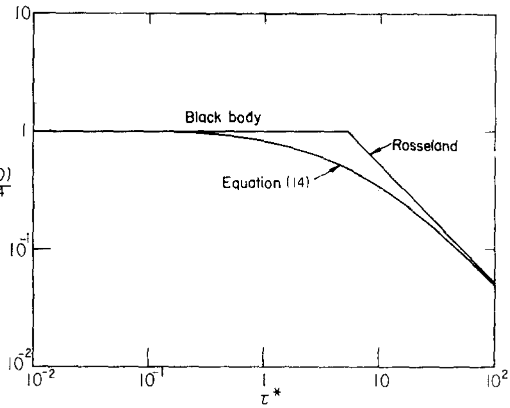

FIG. 2. Radiant heat flux in the inviscid incompressible stagnation flow.

The solution to equations (10) and (11) yields the radiative heat flux to the cold wall as

$$q^R(0) = -2\left(\frac{16Bo}{3\pi\tau_L}\right)^{1/2}\sigma T_i^4\left[1 + 2\,C\,k_{Ri}\left(\frac{3Bo}{16\pi\tau_L}\right)^{1/2}\right]^{-1} \qquad (12)$$

Matching equation (11) with the black-body results yields a value of $C = 16/3\,k_{Ri}$.

According to Goulard [1], a representative thermal-layer thickness can be defined as

$$\delta^* = \frac{1}{2}\left(\frac{16\pi\sigma T_i^3}{3\rho C_p a k_{Ri}}\right)^{1/2} = \frac{L}{2}\left(\frac{16\pi}{3Bo\tau_L}\right)^{1/2} \qquad (13)$$

Equation (12) can then be written as

$$q^R(0) = \frac{-16\sigma T_i^4}{3\tau^*}\left[1 + 16/3\tau^*\right]^{-1} \qquad (14)$$

where $\tau^* = k_{Ri}\,\delta^*$. The above result is shown in Fig. 2.

REFERENCES

1. R. GOULARD, The transition from black-body to Rosseland formulations in optically thick flows *Int. J. Heat Mass Transfer* **6**, 927–930 (1963).
2. S. N. SHORIN, Heat exchange by radiation in the presence of an absorbing medium, *Izv. Akad. Nauk SSSR*, No. 3, 389–406 (1951). See also abridged translation in *Engrs Digest* **12**, 324–328 (1951).
3. R. G. DEISSLER, Diffusion approximation for thermal radiation in gases with jump boundary condition, *ASME* Paper No. 63-*HT*-13 (1963).
4. R. F. PROBSTEIN, Radiation slip, *AIAA J.* **1**, 1202–1204 (1963).
5. E. H. KENNARD, *Kinetic Theory of Gases*, p. 315. McGraw-Hill, New York (1938).